

INEQUALITIES AND REGIONS

- Diagrams can be drawn to represent inequalities.
- The main steps involved are
 - Draw the boundary line
 - Use a solid line if the inequality is a $\leq$ or a $\geq$.
 - Use a dotted line if the inequality is a $<$ or a $>$.
 - Determine the which side of the boundary is needed.
 - Shade out the unwanted region.

Examples

Draw the diagram to represent the inequality $x \geq 2$.

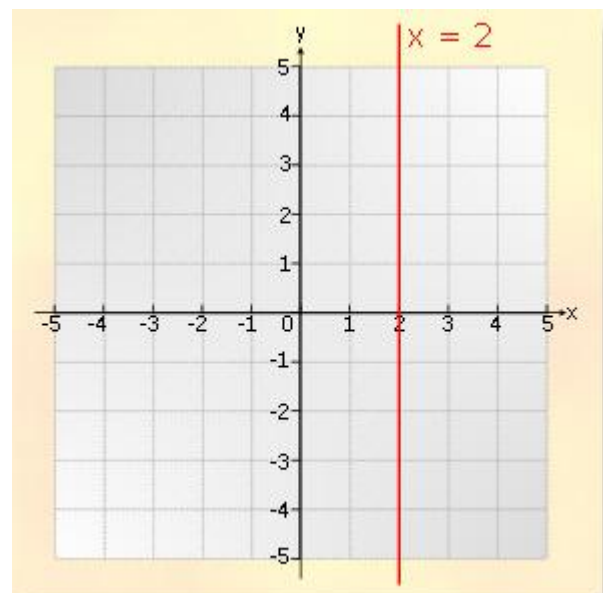
Solution

- Draw the boundary line $x = 2$

This line represents all points for which $x = 2$.

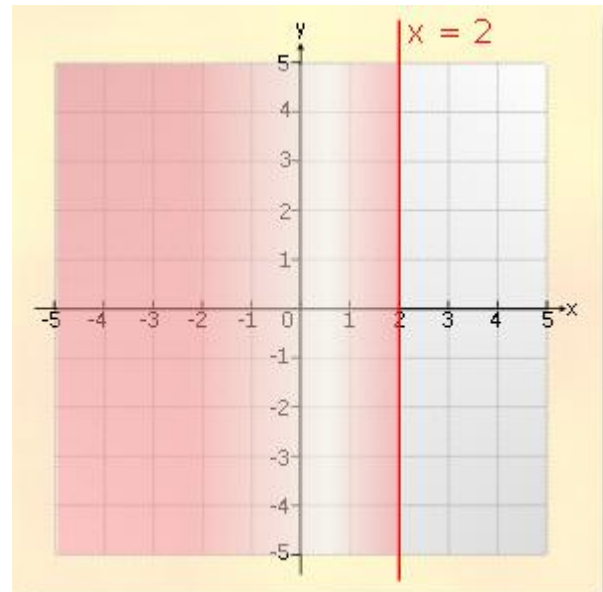
The region to the right contains all points whose x coordinates are greater than 2.

This is the side we want.



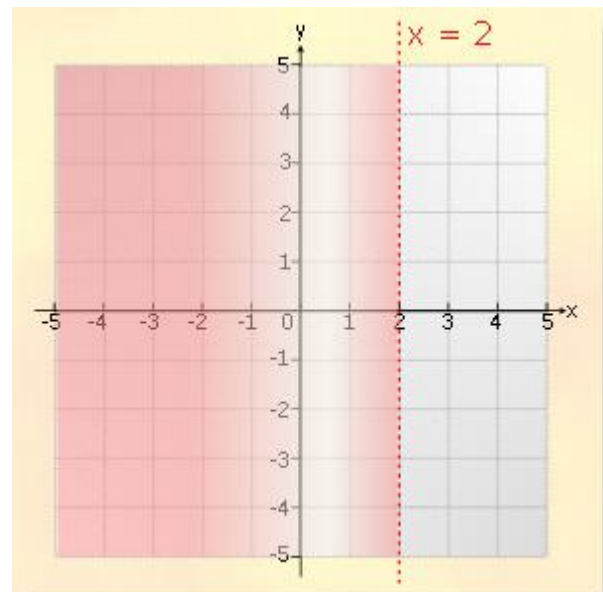
- Shade the unwanted region.

In this case, the boundary line is solid because we want to include $x = 2$.

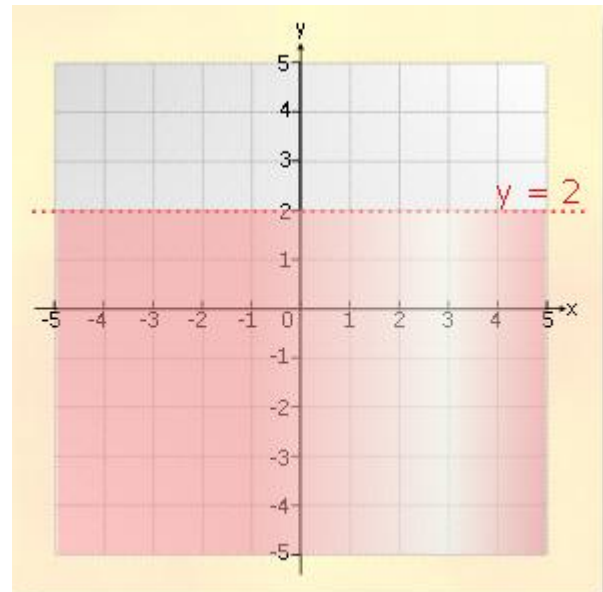


This is the region $x > 2$.

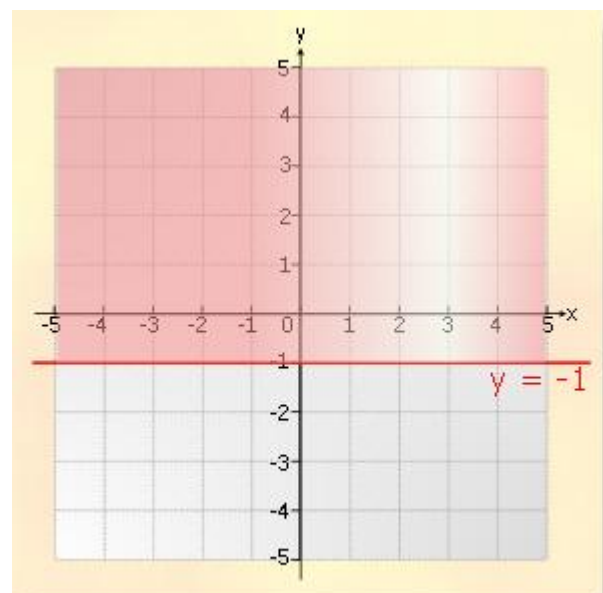
The boundary line is dotted because we don't want to include $x = 2$.



This diagram represents the inequality $y > 2$.



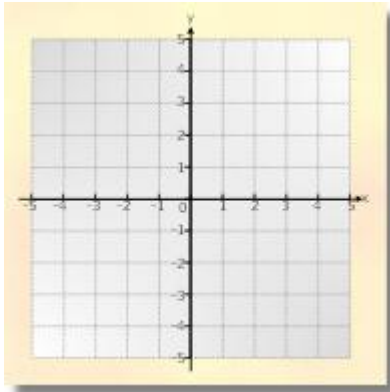
This diagram represents the inequality $y \leq -1$.



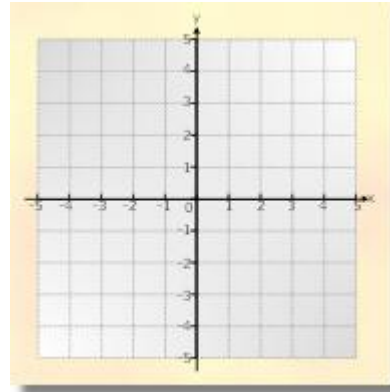
EXERCISE 1

Draw diagrams to represent the following inequalities.

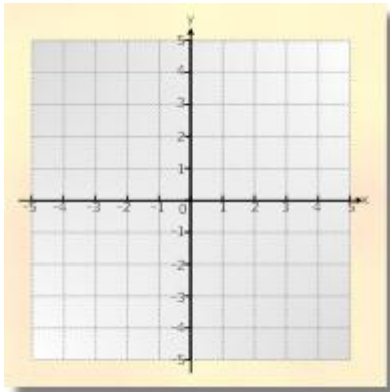
1. $x \geq 2$



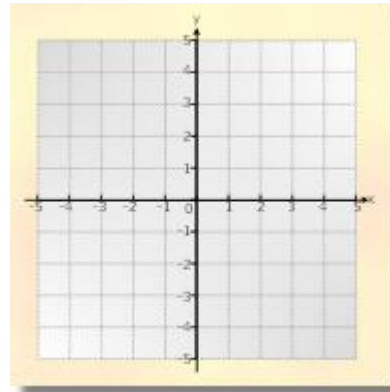
2. $y \leq 3$



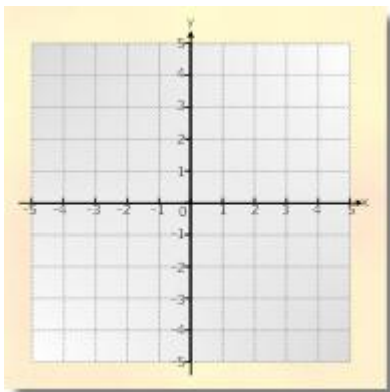
3. $x > -1$



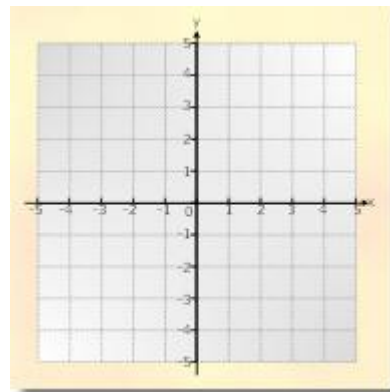
4. $y < 4$



5. $x \geq 0$



6. $x \leq -4$



DOUBLE INEQUALITIES

- Diagrams can be drawn to represent double inequalities.
- Draw both boundary lines and shade out the unwanted regions.

Example

- Draw the region to represent $-3 < x < 2$.
- State whether or not the points $(1, 1)$ and $(-4, 2)$ lie in the given region.

Solution

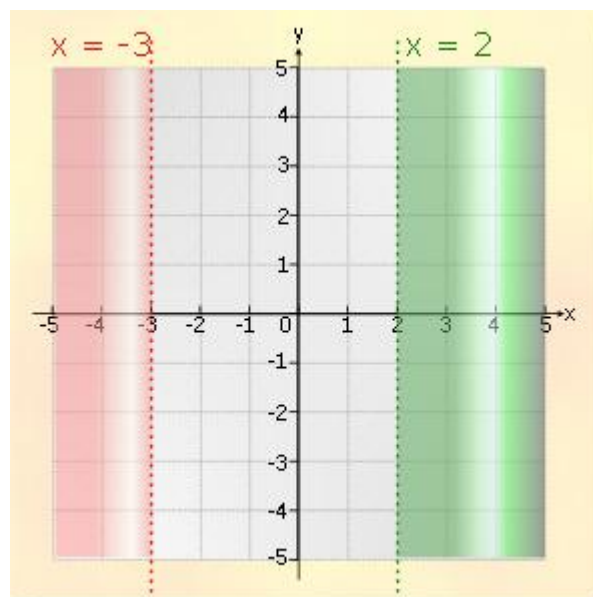
- $-3 < x < 2$ gives two inequalities
 $x > -3$ and $x < 2$

The boundary lines are
 $x = -3$ and $x = 2$.

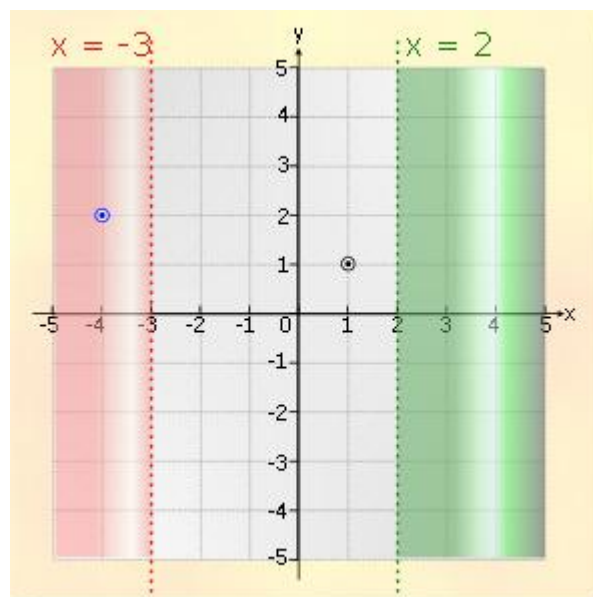
Dotted lines are used because neither $x = 2$ nor $x = -3$ are wanted.

The boundary lines are drawn separately and the unwanted regions shaded.

The unshaded region represents
 $-3 < x < 2$.



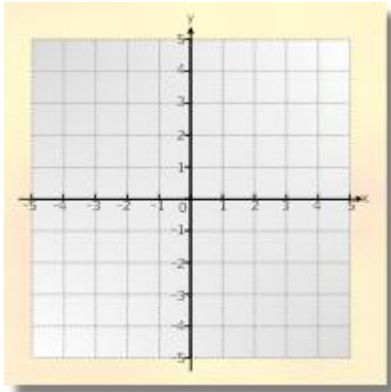
- Plot the point $(1, 1)$
 It lies in the region.
 Plot the point $(-4, 2)$
 It lies outside of the given region.



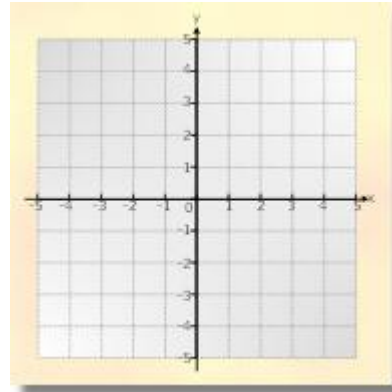
EXERCISE 2

Draw diagrams to represent the following inequalities.

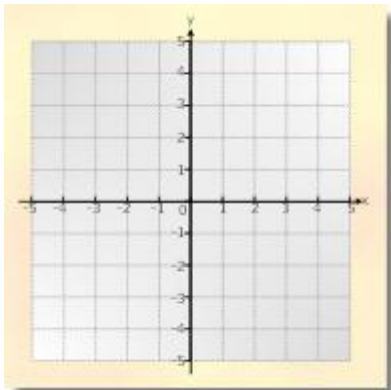
1. $2 \leq x \leq 4$



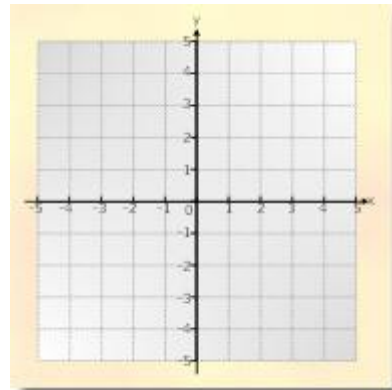
2. $-3 < x < 1$



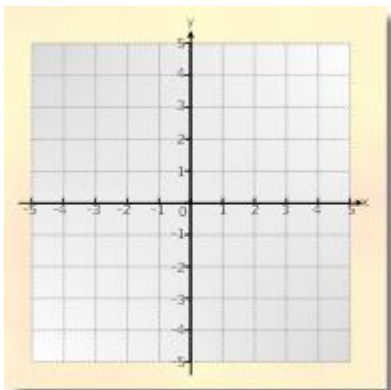
3. $0 \leq x < 4$



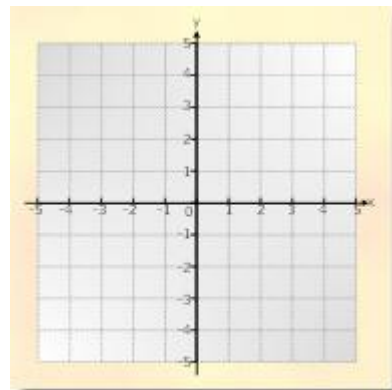
4. $-3 < y < 1$



5. $-2 < y \leq 3$



6. $3 \leq x < 5$



DOUBLE INEQUALITIES**Example**

Draw a diagram to represent the region defined by the inequalities $-1 \leq x \leq 2$ and $-3 \leq y \leq 0$

Solution

There are four boundary lines;

$$x \geq -1,$$

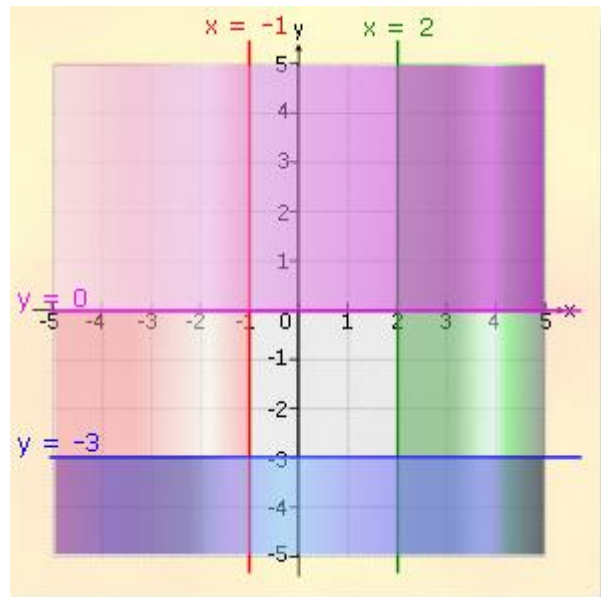
$$x \leq 2,$$

$$y \geq -3 \text{ and}$$

$$y \leq 0.$$

Each boundary line is drawn and the unwanted region shaded.

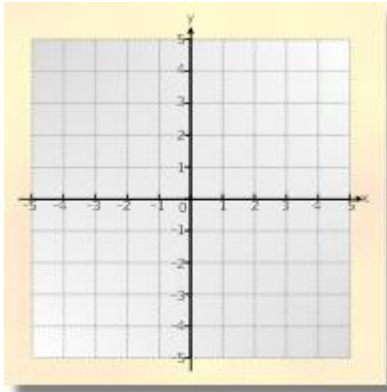
The region which is unshaded represents the inequalities.



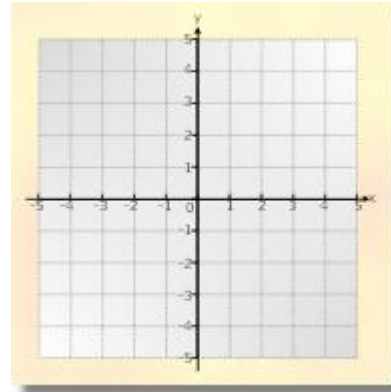
EXERCISE 3

Draw diagrams to represent the following inequalities.

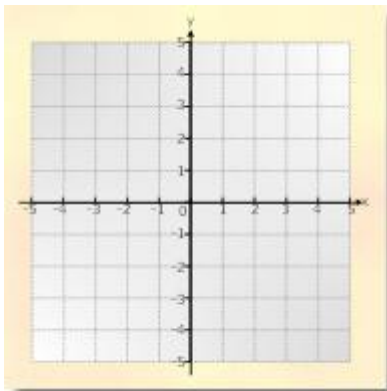
1. $2 \leq x \leq 4, 1 \leq y \leq 3$



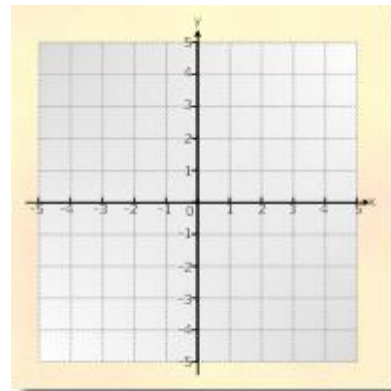
2. $-2 \leq x \leq 2, -2 \leq y < 2$



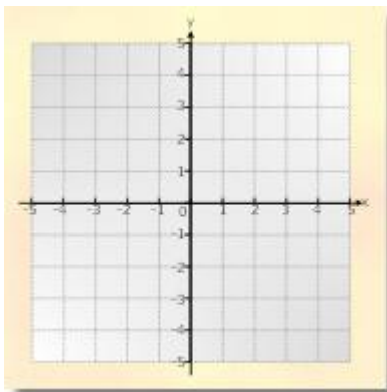
3. $-3 < x \leq 2, y \geq -1$



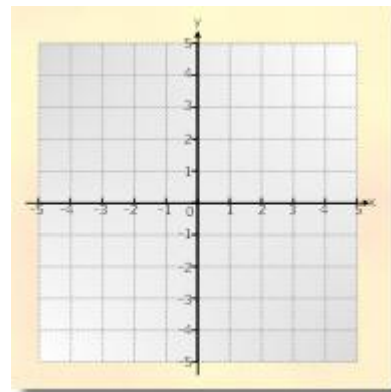
4. $0 \leq x \leq 4, 0 \leq y \leq 3$



5. $-4 \leq x \leq 0, -2 < y \leq 2$



6. $-3 \leq x, 0 \leq y \leq 1$



TWO VARIABLE REGIONS

- So far, the boundary lines have either been
 - Parallel to the x – axis or
 - Parallel to the y – axis.
- Two variable inequalities involve both x and y and are not parallel to the axes.

Example 1

Draw the diagram to represent the inequality $x + y \geq 4$

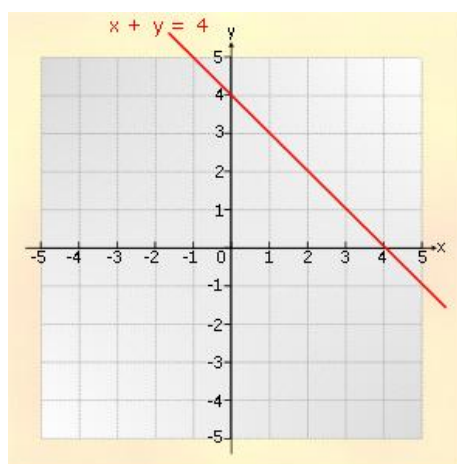
SOLUTION

Step 1

Draw the boundary line $x + y = 4$.

This line represents all the points for which $x + y = 4$.

A solid line is used since it is included in the region.



Step 2

Shade the unwanted region.

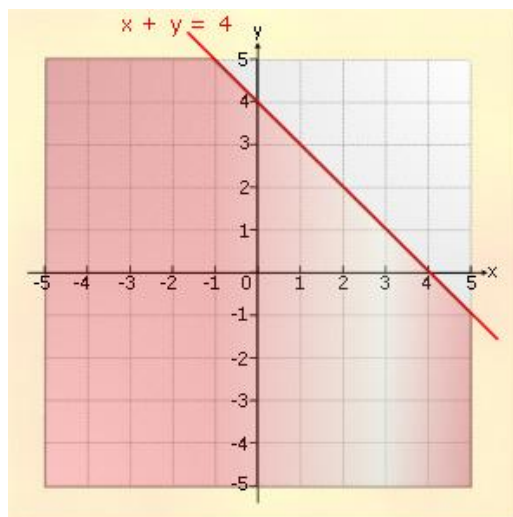
The easiest way to find which side of the line is needed is to test one point.

Test the point $(0, 0)$.

At this point, $x = 0$ and $y = 0$.

$$x + y = 0$$

This is not greater than or equal to 4, so the side containing $(0, 0)$ is not wanted.

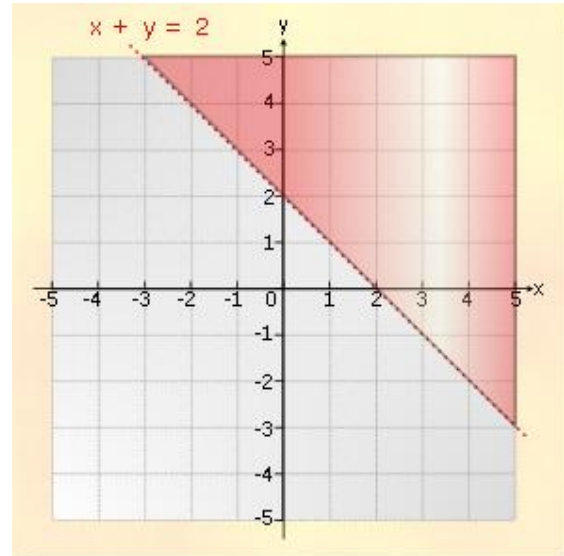


Example 2

Draw a diagram to represent the inequality $x + y < 2$

Solution

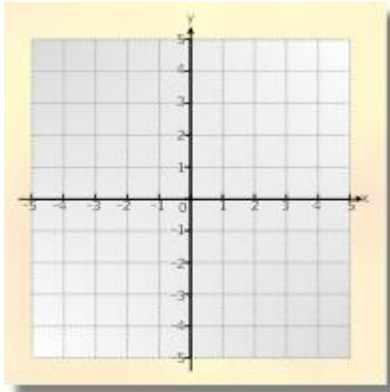
- Boundary line is $x + y = 2$.
- A dotted line is used.
- Test point $(0, 0)$.
- $0 + 0 = 0$.
- This is less than 2 so the side containing the test point is wanted.
- Shade the other side.



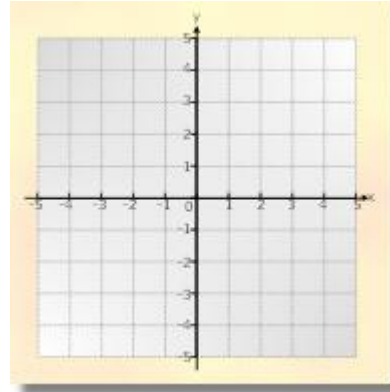
EXERCISE 4

Draw diagrams to represent the following inequalities.

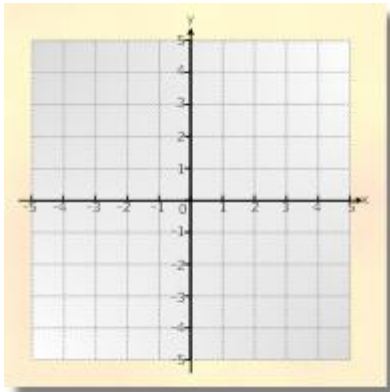
1. $x + y \leq 4$



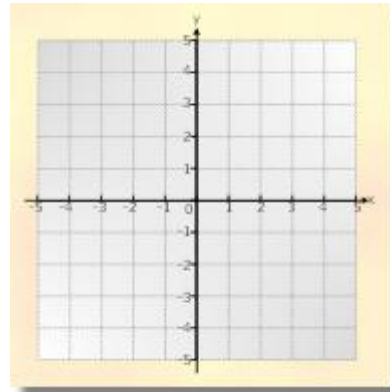
2. $x + y > 3$



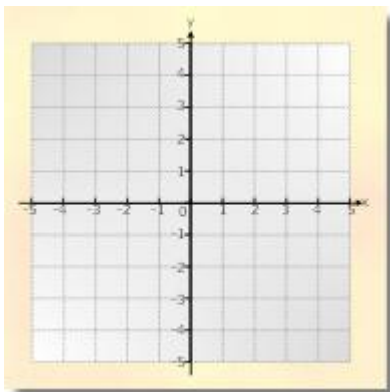
3. $x + y \leq -2$



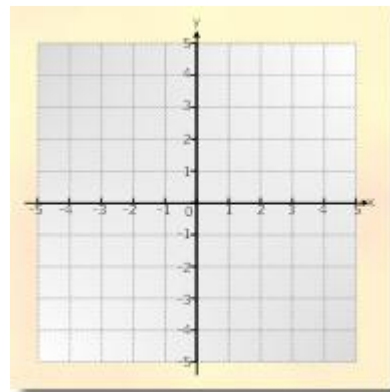
4. $y \leq x$



5. $y - 2x \leq 0$



6. $y + 2x \leq 3$



MULTIPLE BOUNDARIES

- At Higher Tier, you will have to draw a diagram to represent multiple two variable inequalities.
- Take each inequality in turn, shade the unwanted region.
- Make sure that the required region is clearly marked.

Example 1

Draw a diagram to represent the region defined by the inequalities:

$$2x + y \leq 4$$

$$y - 2x < 2$$

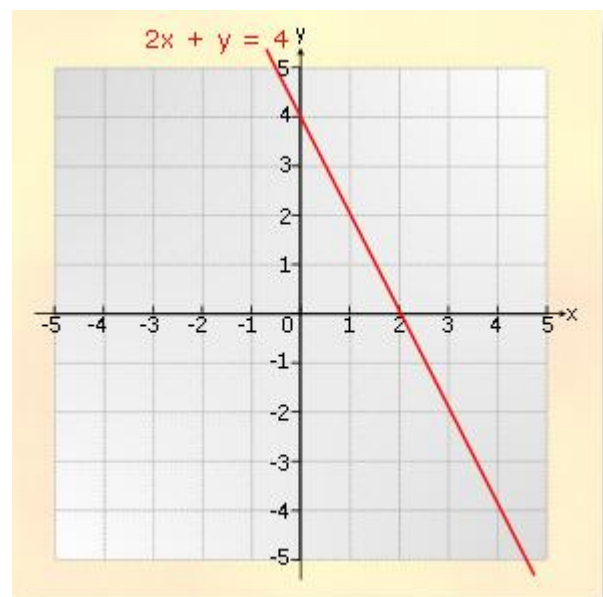
$$x \geq 1$$

Solution

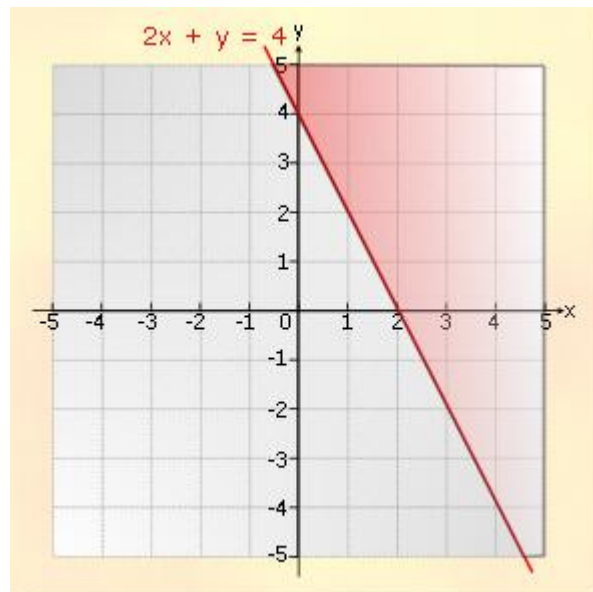
First inequality

- Boundary line is $2x + y = 4$.
- Table of values:

x	0	2
y	4	0



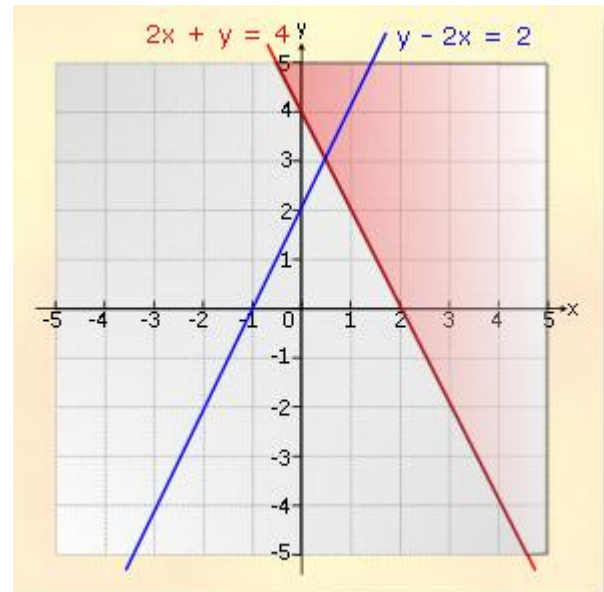
- Test point $(0, 0)$.
- $2 \times 0 + 0 = 0$.
- This is less than 4, so $(0, 0)$ is in the required region.
- Shade out the other side of the boundary.



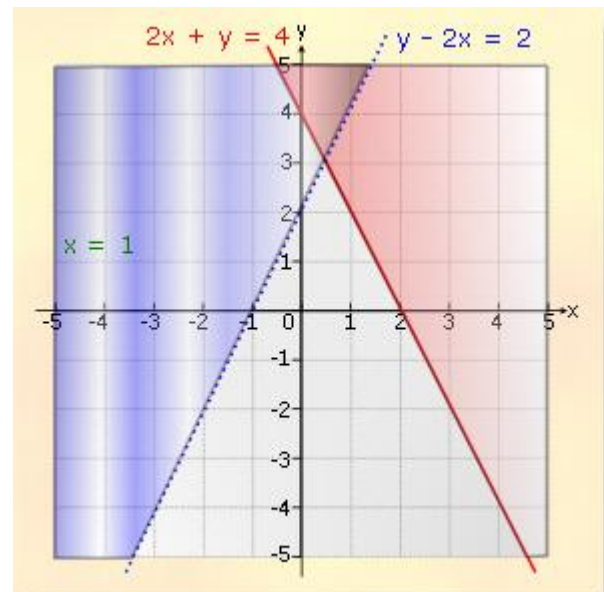
Second inequality

- Boundary line is $y - 2x = 2$.
- Table of values:

x	0	-1
y	2	0

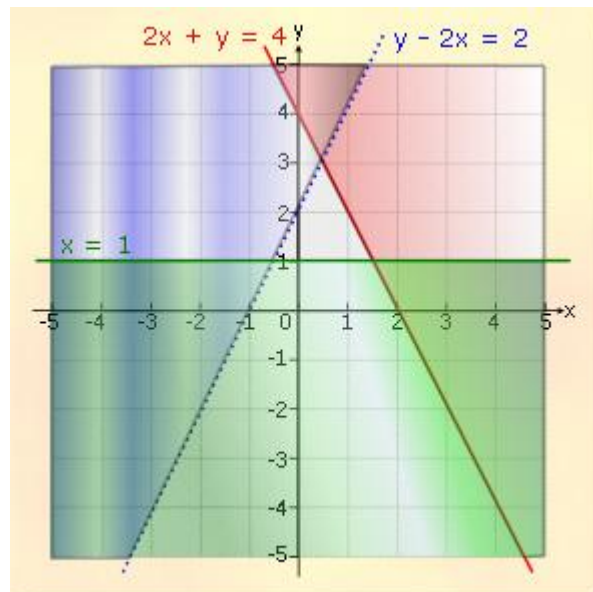


- Test point $(0, 0)$.
- $0 - 2 \times 0 = 0$.
- This is less than 2, so $(0, 0)$ is in the required region.
- Shade out the other side of the boundary.

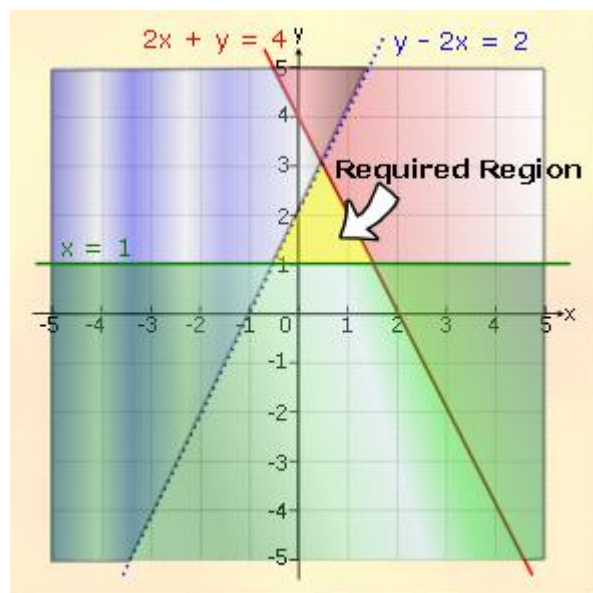


Third inequality

- Boundary line is $x = 1$.
- Shade unwanted region.



- Make sure that the correct region is clearly marked.



Example 2 – A GCSE Question

- a) Draw a diagram to clearly indicate the region which satisfies all of the following inequalities. [4]

$$\begin{aligned} y &< x \\ y &\leq -2x + 5 \\ x &> 1 \\ y &\geq -1 \end{aligned}$$

- b) Write down the coordinates of all the points whose coordinates are integers and lie in the region which satisfies all the inequalities given in (a). [3]

Solution

a)

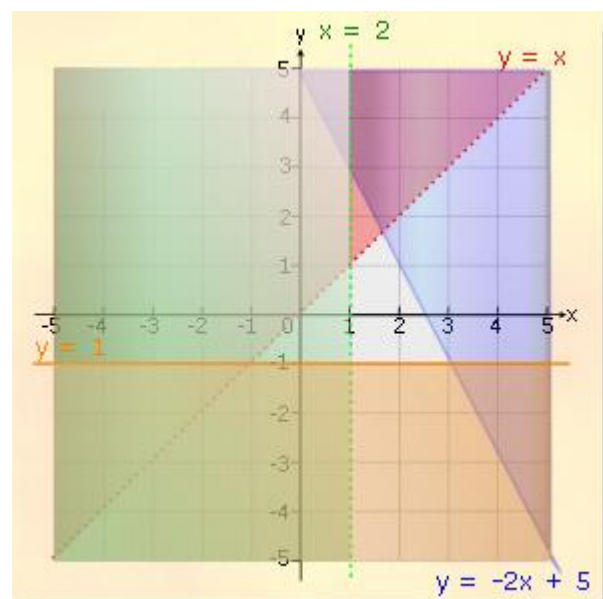
Boundary lines are:

$$y = x$$

$$y = -2x + 5$$

$$x = 1$$

$$y = -1$$



b)

The points marked are the only whole number (integer) coordinates inside the region.

Those on a solid line are okay.

Those on a dotted line are not included.

The points are

$(2, -1)$, $(2, 0)$, $(2, 1)$ and $(3, -1)$

