INEQUALITIES AND REGIONS

- Diagrams can be drawn to represent inequalities.
- The main steps involved are
  - Draw the boundary line
    - Use a solid line if the inequality is a ≤ or a ≥.
    - Use a dotted line if the inequality is a < or a >.
  - Determine which side of the boundary is needed.
  - Shade out the unwanted region.

Examples

Draw the diagram to represent the inequality x ≥ 2.

Solution

- Draw the boundary line x = 2
  - This line represents all points for which x = 2.
  - The region to the right contains all points whose x coordinates are greater than 2.
  - This is the side we want.
Shade the unwanted region.

**In this case, the boundary line is solid because we want to include x = 2.**

This is the region x > 2.

**The boundary line is dotted because we don’t want to include x = 2.**
This diagram represents the inequality \( y > 2 \).

This diagram represents the inequality \( y \leq -1 \).
EXERCISE 1

Draw diagrams to represent the following inequalities.

1. $x \geq 2$

2. $y \leq 3$

3. $x > -1$

4. $y < 4$

5. $x \geq 0$

6. $x \leq -4$
DOUBLE INEQUALTIES

- Diagrams can be drawn to represent double inequalities.
- Draw both boundary lines and shade out the unwanted regions.

Example

a) Draw the region to represent \(-3 < x < 2\).

b) State whether or not the points (1, 1) and (-4, 2) lie in the given region.

Solution

a) \(-3 < x < 2\) gives two inequalities
   \[ x > -3 \text{ and } x < 2 \]

   The boundary lines are
   \[ x = -3 \text{ and } x = 2. \]

   Dotted lines are used because neither \(x = 2\) nor \(x = -3\) are wanted.
   The boundary lines are drawn separately and the unwanted regions shaded.
   The unshaded region represents \(-3 < x < 2\).

b) Plot the point (1, 1)
   It lies in the region.

   Plot the point (-4, 2)
   It lies outside of the given region.
EXERCISE 2

Draw diagrams to represent the following inequalities.

1. $2 \leq x \leq 4$

2. $-3 < x < 1$

3. $0 \leq x < 4$

4. $-3 < y < 1$

5. $-2 < y \leq 3$

6. $3 \leq x < 5$
**DOUBLE INEQUALITIES**

**Example**

Draw a diagram to represent the region defined by the inequalities $-1 \leq x \leq 2$ and $-3 \leq y \leq 0$

**Solution**

There are four boundary lines;

- $x \geq -1,$
- $x \leq 2,$
- $y \geq -3$ and
- $y \leq 0.$

Each boundary line is drawn and the unwanted region shaded.

The region which is unshaded represents the inequalities.
**EXERCISE 3**

Draw diagrams to represent the following inequalities.

1. \(2 \leq x \leq 4, \ 1 \leq y \leq 3\)

2. \(-2 \leq x \leq 2, \ -2 \leq y < 2\)

3. \(-3 < x \leq 2, \ y \geq -1\)

4. \(0 \leq x \leq 4, \ 0 \leq y \leq 3\)

5. \(-4 \leq x \leq 0, \ -2 < y \leq 2\)

6. \(-3 \leq x, \ 0 \leq y \leq 1\)
TWO VARIABLE REGIONS

- So far, the boundary lines have either been
  - Parallel to the x-axis or
  - Parallel to the y-axis.
- Two variable inequalities involve both x and y and are not parallel to the axes.

**Example 1**
Draw the diagram to represent the inequality \( x + y \geq 4 \)

**Solution**

**Step 1**
Draw the boundary line \( x + y = 4 \).
This line represents all the points for which \( x + y = 4 \).
A solid line is used since it is included in the region.

**Step 2**
Shade the unwanted region.
The easiest way to find which side of the line is needed is to test one point.
Test the point \((0, 0)\).
At this point, \( x = 0 \) and \( y = 0 \).
\( x + y = 0 \)
This not greater than or equal to 4, so the side containing \((0, 0)\) is not wanted.
Example 2
Draw a diagram to represent the inequality $x + y < 2$

Solution
- Boundary line is $x + y = 2$.
- A dotted line is used.
- Test point (0, 0).
- $0 + 0 = 0$.
- This is less than 2 so the side containing the test point is wanted.
- Shade the other side.
EXERCISE 4

Draw diagrams to represent the following inequalities.

1. $x + y \leq 4$
2. $x + y > 3$
3. $x + y \leq -2$
4. $y \leq x$
5. $y - 2x \leq 0$
6. $y + 2x \leq 3$
MULTIPLE BOUNDARIES

- At Higher Tier, you will have to draw a diagram to represent multiple two variable inequalities.
- Take each inequality in turn, shade the unwanted region.
- Make sure that the required region is clearly marked.

Example 1

Draw a diagram to represent the region defined by the inequalities:

\[ 2x + y \leq 4 \]
\[ y - 2x < 2 \]
\[ x \geq 1 \]

Solution

First inequality

- Boundary line is \( 2x + y = 4 \).
- Table of values:

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>
• Test point (0, 0).
• $2 \times 0 + 0 = 0$.
• This is less than 4, so (0, 0) is in the required region.
• Shade out the other side of the boundary.
Second inequality

- Boundary line is $y - 2x = 2$.
- Table of values:

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

- Test point $(0, 0)$.
- $0 - 2 \times 0 = 0$.
- This is less than 2, so $(0, 0)$ is in the required region.
- Shade out the other side of the boundary.
Third inequality

- Boundary line is \( x = 1 \).
- Shade unwanted region.

- Make sure that the correct region is clearly marked.
Example 2 – A GCSE Question

a) Draw a diagram to clearly indicate the region which satisfies all of the following inequalities. [4]

\[ y < x \]
\[ y \leq -2x + 5 \]
\[ x > 1 \]
\[ y \geq -1 \]

b) Write down the coordinates of all the points whose coordinates are integers and lie in the region which satisfies all the inequalities given in (a). [3]

Solution

a) Boundary lines are:

\[ y = x \]
\[ y = -2x + 5 \]
\[ x = 1 \]
\[ y = -1 \]
b) The points marked are the only whole number (integer) coordinates inside the region.

Those on a solid line are okay.
Those on a dotted line are not included.
The points are
(2, –1), (2, 0), (2, 1) and (3, –1)