TRANSFORMATION OF GRAPHS

TRANSLATIONS

- The graph of $y = x^2$ is drawn in red.
- $y = x^2 + 2$ has the same shape, but it has been shifted up the $y$-axis.
- $y = x^2 - 1$ has the same shape, but it has been shifted down the $y$-axis.

In general, 
If $y = f(x)$ then 
$y = f(x) + a$
shifts the graph up/down the $y$-axis by $a$ units.

- The graph of $y = x^2$ is drawn in red.
- $y = (x + 2)^2$ has the same shape, but it has been shifted along the $x$-axis.

In general, 
If $y = f(x)$ then 
$y = f(x + a)$
shifts the graph left/right along the $x$-axis by $a$ units.
**REFLECTIONS**

- The graph of \( y = x^2 \) is drawn in red.
- \( y = -x^2 \) has the same shape, but it has been reflected about the \( x \)-axis.

**In general,**

If \( y = f(x) \) then
\[ y = -f(x) \]

is a reflection in the \( x \)-axis.

- The graph of \( y = x + 2 \) is drawn in red.
- \( y = -x + 2 \) has been reflected about the \( y \)-axis.

**In general,**

If \( y = f(x) \) then
\[ y = f(-x) \]

is a reflection in the \( y \)-axis.
**Enlargements**

- The graph of $y = x^2$ is drawn in red.
- $y = 4x^2$ has been enlarged by a factor of 4 in the $y$-direction.

**In general,**

If $y = f(x)$ then $y = af(x)$ is an enlargement by a factor of $a$ in the $y$-direction.
Graphical software Exercise

Draw each group using the same axes. The original graph is the first in the list. Note the effect on the original graph.

**Translations**
1. \( y = x \), \( y = x + 2 \), \( y = x - 1 \)
2. \( y = x^2 \), \( y = x^2 + 1 \), \( y = x^2 - 3 \)
3. \( y = x^3 \), \( y = x^3 + 3 \), \( y = x^3 - 1 \)
4. \( y = x^2 \), \( y = (x + 3)^2 \), \( y = (x - 2)^2 \)
5. \( y = x^3 \), \( y = (x+1)^3 \), \( y = (x -2)^3 \)

**Reflections**
1. \( y = x \), \( y = -x \)
2. \( y = x^2 \), \( y = -x^2 \)
3. \( y = (x + 2)^2 \), \( y = -(x + 2)^2 \)
4. \( y = x^2 + x \), \( y = (-x)^2 + (-x) \)
5. \( y = x^3 + x^2 \), \( y = (-x)^3 + (-x)^2 \)

**Enlargements**
1. \( y = x^2 \), \( y = 2x^2 \), \( y = 3x^2 \)
2. \( y = (x + 1)^3 \), \( y = 2(x + 1)^3 \)
3. \( y = x^2 \), \( y = (2x)^2 \)
4. \( y = 3x^2 + x \), \( y = 3(2x)^3 + (2x) \)

**Combinations**
1. \( y = x^2 \), \( y = y - 1 \) = \( x^2 \), \( y - 1 \) = \( (x + 2)^2 \)
2. \( y = x^2 \), \( y + 2 \) = \( x^2 \), \( y + 2 \) = \( 3x^2 \)
3. \( y = x^2 \), \( y + 2 \) = \( x^2 \), \( y + 2 \) = \( 3(x - 1)^2 \)