

PERCENTAGES

INTRODUCTION

- Percentages are used widely in everyday life and you should be confident in their use!



- Percentages are fractions with 100 as the denominator.

EXAMPLE

$$75\% = \frac{75}{100} = \frac{3}{4}$$

- You should know the following percentage / fraction equivalents:

$$100\% = 1$$

$$75\% = \frac{3}{4}$$

$$50\% = \frac{1}{2}$$

$$25\% = \frac{1}{4}$$

$$20\% = \frac{1}{5}$$

$$10\% = \frac{1}{10}$$

- To change a fraction or decimal into a percentage, simply multiply by 100.

Examples

$$0.5 = 0.5 \times 100\% = 50\%$$

$$\frac{3}{4} = \frac{3}{4} \times 100\% = 75\%$$

PERCENTAGE OF A QUANTITY

- Remember, the 'of' means multiply.

EXAMPLE

Find 30% of 500

ANSWER

$$\begin{aligned} 30\% \text{ of } 500 &= 30\% \times 500 \\ &= \frac{30}{100} \times 500 \end{aligned}$$

If this is on a **calculator** paper,

- key in $30 \times 500 \div 100 = 150$

If this is on a **non - calculator** paper,

- work out 10% of 500 first, by dividing by 10,
- 10% of 500 = 50
- then multiply by 3 to give 30%
- 30% of 500 = $3 \times 50 = 150$

- In a non-calculator question, complicated looking percentages can be found by working out simpler percentages

EXAMPLE

Find 17.5% of 260

Answer

$$10\% \text{ of } 260 = 26$$

$$5\% \text{ of } 260 = 13$$

$$2.5\% \text{ of } 260 = 6.50$$

$$\begin{aligned} \therefore 17.5\% \text{ of } 260 &= 26 + 13 + 6.50 \\ &= 45.50 \end{aligned}$$

EXAMPLE

Find 2.5% of £135

Answer

$$2.5\% \text{ of } £135 = 2.5\% \times £135$$

$$= \frac{2.5}{100} \times 135 = £3.375 = £3.38$$

(This is a money question so round the answer sensibly)

PERCENTAGE CHANGE

- Quantities change – prices may go up or may go down, wages increase or decrease etc.
- Quantities often change by a percentage.

$$\text{Percentage change} = \frac{\text{change}}{\text{original}} \times 100\%$$

EXAMPLE 1

A DVD player costs £140. In a sale it is reduced to £91. What is the percentage reduction?

ANSWER

$$\text{Reduction} = £140 - £91 = £49 \quad (\text{this is the change})$$

$$\begin{aligned} \% \text{ reduction} &= \frac{49}{140} \times 100\% \\ &= 35\% \end{aligned}$$

EXAMPLE 2

Sara bought a house for £56000. Three years later, she sold it for £72800. What was the percentage profit?

ANSWER

$$\text{Profit} = £72800 - £56000 = £16800 \quad (\text{this is the change})$$

$$\begin{aligned} \% \text{ profit} &= \frac{16800}{56000} \times 100\% \\ &= 30\% \end{aligned}$$

ONE QUANTITY AS A PERCENTAGE OF ANOTHER

- We often need to express quantities as percentages, for example the percentage of people unemployed, the percentage of students gaining a grade A to C in maths etc.
- To express one quantity as a percentage of another, first form the fraction then multiply by 100% to change it into a percentage.

EXAMPLE

Julian scored 45 out of 60 in a test. What percentage is this?

ANSWER

$$\begin{aligned}45 \text{ out of } 60 &= \frac{45}{60} \\ &= \frac{45}{60} \times 100\% \\ &= 75\%\end{aligned}$$

EXAMPLE

In a school, 68 out of 93 students got 5 GCSEs at grade A* to C. What percentage is this ?

ANSWER

$$\begin{aligned}58 \text{ out of } 93 &= \frac{58}{93} \\ &= \frac{58}{93} \times 100\% \\ &= 62.4\% \quad (\text{to 1 d.p.})\end{aligned}$$

EXAMPLE

Express the first quantity as a percentage of the second:

- 20 cm, 70 cm
- 4 mm, 4 cm

ANSWER

a. 20 cm as a percentage of 70 cm = $\frac{20}{70} \times 100 = 28.57\%$

- b. First express both quantities in the same unit, in this case change cm to mm.

$$4 \text{ cm} = 4 \times 10 \text{ mm} = 40 \text{ mm}$$

$$\therefore 4 \text{ mm as a percentage of } 4 \text{ cm} = \frac{4}{40} \times 100 = 10\%$$

PERCENTAGE INCREASE AND DECREASE

- There are many everyday situations when a quantity is increased or decreased by a percentage. The price of goods in a sale may fall by 10%, wages might go up by 4%, etc.

Example

The price of a DVD player was £150. In a sale it is reduced by 15%. What is the sale price?

Answer

<p><u>Method 1</u></p> $15\% \text{ of } £150 = \frac{15}{100} \times 150$ $= £22.50$ <p>Sale price = £150 – £22.50 = £127.50</p>	<p><u>Method 2</u></p> <p>Sale price = 100% – 15% = 85%</p> <p>85% = 0.85</p> <p>Sale price = 0.85 × £150 = £127.50</p>
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Example

Kelly earns £350 a week. She gets a 3% pay rise. How much will she get each week after the pay rise?

Answer

<p><u>Method 1</u></p> <p>3% of £350 = £10.50</p> <p>New wage = £350 + £10.50 = £360.50</p>	<p><u>Method 2</u></p> <p>New wage = 100% + 3% = 103%</p> <p>103% of £350 = £360.50</p>
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Example

A house was worth £65000 last year. During the year it has gone up by 12%. How much is it worth now?

Answer

<p><u>Method 1</u></p> <p>12% of £65000 = £7800</p> <p>New price = £65000 + £7800 = £72800</p>	<p><u>Method 2</u></p> <p>New price = 100% + 12% = 112%</p> <p>112% of £65000 = £72800</p>
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