

DIRECT PROPORTION

- If two quantities increase at the same rate, for example, if one doubles so does the other, then the quantities are said to be directly proportional.

y is proportional to x

we write: $y \propto x$

this means: $y = kx$

where k is a positive number.

- If we know one pair of values for x and y, k can be found.

EXAMPLE 1

Given that y is proportional to x and that when $x = 7$, $y = 14$, find the relationship between x and y.

SOLUTION

$$y \propto x$$

$$\therefore y = kx$$

$$\text{when } x = 7, y = 14$$

$$\therefore 14 = k \times 7$$

$$\therefore k = 2$$

$$\therefore \text{The relationship between } x \text{ and } y \text{ is: } y = 2x$$

EXAMPLE 2

The current, I, in a particular type of wire is proportional to the voltage, V.

If $I = 10$ amps when $V = 7$ volts, find

- a) The current when the voltage is 20 volts.
- b) The voltage when the current is 12 amps.

SOLUTION

- Step 1 Change the sentence into a proportionality using the \propto symbol. $I \propto V$
- Step 2 Replace the \propto with $= k$ to make an equation. $I = kV$
- Step 3 Use the values given to find k . In this case, $I = 10$ when $V = 7$.
 $10 = 7k$
 $\therefore k = \frac{10}{7}$
- Step 4 Use the value of k to rewrite the equation. $I = \frac{10}{7}V$
- Step 5 Now answer the questions using this equation.
- a) When $V = 20$
 $I = \frac{10}{7} \times 20 = 28.6$ amps
- b) When $I = 12$
 $12 = \frac{10}{7} \times V$
 $\therefore V = 12 \times \frac{7}{10} = 8.4$ volts

VARIATION

- The same principle as used in direct proportion can be extended. The following statements are frequently used at GCSE.

a is proportional to the square of b
 c is proportional to the square root of d
 p varies as the cube of q

- Each statement can be written as a proportionality and as an equation:

Statement	Proportionality	Equation
a is proportional to b	$a \propto b$	$a = kb$
a is proportional to the square of b	$a \propto b^2$	$a = kb^2$
a is proportional to the cube of b	$a \propto b^3$	$a = kb^3$
a varies as the square root of b	$a \propto \sqrt{b}$	$a = k\sqrt{b}$

- k is called the constant of proportion or variation.
- Many type of proportionality have already been met,

Circumference is proportional to the diameter.	$c \propto d$	$c = \pi d$
Area of a circle is proportional to the square of the radius.	$A \propto r^2$	$A = \pi r^2$
Force is proportional to acceleration.	$F \propto a$	$F = ma$

Example

The value (V) of a diamond varies directly as the square of its weight (w). A diamond weighing 5.0 g is worth £2500. How heavy is a diamond that is valued at £6000 ?

**Solution**

- | | | |
|--------|--|---|
| Step 1 | Change the sentence into a proportionality using the \propto symbol. | $V \propto w^2$ |
| Step 2 | Replace the \propto with $= k$ to make an equation. | $V = kw^2$ |
| Step 3 | Use the values given to find k . In this case, $V = 2500$ when $w = 5$. | $2500 = k \times (5)^2$
$\therefore 2500 = 25k$
$\therefore k = 100$ |
| Step 4 | Use the value of k to rewrite the equation. | $V = 100w^2$ |
| Step 5 | Now answer the questions using this equation. | $6000 = 100w^2$
$w^2 = 60$
$w = \sqrt{60}$
$w = 7.75 \text{ g (2 d.p.)}$ |

INVERSELY PROPORTION (INDIRECT PROPORTION)

- If two quantities increase at opposite rates, so that if one doubles the other is halved, the quantities are said to be inversely proportional.

For example,

if it takes **1** man **2** days to dig a trench,
then **2** men should take **1** day
and **4** men should take **½** a day etc...

- The statement y is inversely proportional to x

is written as: $y \propto \frac{1}{x}$

which means: $y = \frac{k}{x}$

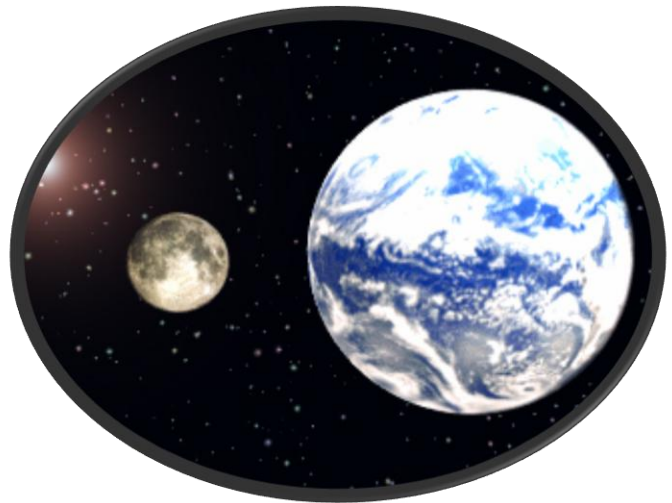
- Many famous statements use inverse proportionality,

Newton's Law of Gravity

The force of attraction between two bodies is inversely proportional to the square of the distance between them.

In other words

$$F \propto \frac{1}{d^2}$$



EXAMPLE

The intensity of light on an object is inversely proportional to the square of the distance of the object from the light.

If the intensity is 8 units at a distance of 5 m, find the intensity at a distance of 4 m.

SOLUTION

- Step 1 Change the sentence into a proportionality using the \propto symbol. $I \propto \frac{1}{s^2}$
Let I = intensity and s = distance.
- Step 2 Replace the \propto with $= k$ to make an equation. $I = \frac{k}{s^2}$
- Step 3 Use the values given to find k . In this case, $I = 8$ when $s = 5$.
 $8 = \frac{k}{5^2}$
 $\therefore 8 = \frac{k}{25}$
 $\therefore k = 8 \times 25$
 $\therefore k = 200$
- Step 4 Use the value of k to rewrite the equation. $I = \frac{200}{s^2}$
- Step 5 Now answer the questions using this equation. $I = \frac{200}{4^2}$
 $= \frac{200}{16}$
 $= 12.5$ units

EXAMPLE

The volume of gas is inversely proportional to its pressure (Boyle's Law)

- a) Find the relationship between the volume, V , pressure, p and a constant, k .
- b) When $p = 400 \text{ N/m}^2$, $V = 2 \text{ m}^3$. Find k .
- c) Find V when $p = 150 \text{ N/m}^2$.
- d) Find p when $V = 5 \text{ m}^3$.

SOLUTION

$$\text{a) } V \propto \frac{1}{p} \quad \therefore V = \frac{k}{p}$$

$$\text{b) } 2 = \frac{k}{400} \quad \therefore k = 800$$

$$\text{c) } V = \frac{800}{p} \quad \therefore V = \frac{800}{150} = 5\frac{1}{3} \text{ m}^3$$

$$\text{d) } V = \frac{800}{p} \quad \therefore 5 = \frac{800}{p} \quad \therefore p = \frac{800}{5} = 160 \text{ N/m}^2$$